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Geometry I
WS 09/10

http://www.math.tu-berlin.de/~sullivan/L/09W/Geo1/

Exercise Sheet 1

Exercise 1: Distance. (4 pts)

Let x and y be two points in the Euclidean space $\mathbb{E}^n$. Prove that the shortest path between them is the line segment connecting them.

Exercise 2: Spherical circles. (4 pts)

In $\mathbf{S}^2$, define the circle of radius r around a point c as

$$C_r(c) := \left\{x \in \mathbf{S}^2 \mid d(c,x) = r\right\},$$

where $d(\cdot,\cdot)$ is the spherical metric. Show that $C_r(c)$ is the intersection of $\mathbf{S}^2$ with a plane. Find the length of the circle.

Exercise 3: Perpendicular bisector. (4 pts)

Given two points $P \neq Q \in \mathbf{S}^2$, define

$$X := \left\{x \in \mathbf{S}^2 \mid d(x,P) = d(x,Q)\right\}.$$

Show that X is a great circle that intersects any great circle through P and Q orthogonally. What is special about the case $P = -Q$?

Exercise 4: Polar triangle. (4 pts)

For a point $P \in \mathbf{S}^2$, let H_P denote the hemisphere with interior pole P. Show that a point $P \in \mathbf{S}^2$ is contained in a spherical triangle Δ if and only if H_P contains the polar triangle of Δ.

Due: Tutorial on 30.10.09