Geometry I

Practice problems for Test, 2009 December 17

1. Suppose four points $A, B, C, D \in \mathbb{R}P^1$ have cross-ratio $\mathrm{R}(A, B, C, D) = q$. What is the cross-ratio $\mathrm{R}(B, C, A, D)$?

2. Let $\triangle ABC$ be a hyperbolic triangle with angles $\alpha = \pi/6$, $\beta = \pi/4$, $\gamma = \pi/2$. What are the side lengths? What is the area of the triangle?

3. Suppose ℓ, ℓ' are lines in $\mathbb{H}^2$ with unit normal vectors $n = (5/4, 0, 3/4)$ and $n' = (1, 1, -1)$, respectively, in Lorentz space. Do these lines intersect? If so, at what angle and at what point $p \in \mathbb{H}^2$? If not, what is the distance between the lines?

4. Show that there are circles in the hyperbolic plane which cannot be inscribed into a triangle.

5. Suppose U and U' are vector subspaces of V. Show that the projective subspace $P(U + U') \subset P(V)$ is the union of all lines xx' where $x \in P(U)$ and $x' \in P(U')$ with $x \neq x'$.